Problem sheet 10

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1.) Light edges
Show that if an edge (u, v) is contained in some minimum-spanning tree, then it is a light edge crossing some cut of the graph. (2P)

2.) Just light edges?
Give a simple example of a graph such that the set of edges:
$\{(u, v)$: there exists a cut $(S, V - S)$ such that (u, v) is a light edge crossing $(S, V - S)\}$
does not form a minimum spanning tree. (1P)

3.) Prim on adjacency matrix representation
Suppose that the graph $G = (V, E)$ is represented as an adjacency matrix. Give a simple implementation of Prim's Algorithm for this case that runs in $O(V^2)$. (3P)

4.) Fast Kruskal
Suppose that all edge weights in a graph are integers in the range from 1 to $|V|$. How fast can you make Kruskal's Algorithm run? What if the edge weights are integers in the range from 1 to W for some constang W ? (2P)

5.) Fast Prim
Suppose that all edge weights in a graph are integers in the range from 1 to $|V|$. How fast can you make Prim's Algorithm run? What if the edge weights are integers in the range from 1 to W for some constang W ? (2P)