Problem sheet 3

Course 320201 Fundamental Computer Science I, Dr. Holger Kenn
e-mail: h.kenn@iu-bremen.de, tel.:+49 421 200 3112

This problemsheet's solution is to be handed in Friday, September 26th *before the lecture*, either clearly readable on paper or as a *PDF* file via e-mail to h.kenn@iu-bremen.de. (and not to one of the TAs!)

1.) Priority queues

2.1) Define a delete operation, comparable to the insert operation, that deletes an element $A[i]$ from a priority queue and that runs in $O(\lg n)$.

(2p)

2.) Hoare partition correctness

The version of PARTITION given in chapter 7 in Cormen/Leiserson/Rivest/Stoll is not the original partitioning algorithm. Here is the original algorithm, which is due to T.Hoare: HOARE-PARTITION (A,p,r)

```
x ← A[p]
i ← p − 1
j ← r + 1
while true do
    repeat
        j ← j − 1
    until A[j] ≤ x
    repeat
        i ← i + 1
    until A[i] ≥ x
    if i < j then
        EXCHANGE  A[i] ↔ A[j]
    else
        return j
    end if
end while
```

2.1) Demonstrate the operation of HOARE-PARTITION on the array $A = \{13, 19, 9, 5, 12, 8, 7, 4, 11, 2, 6, 21\}$, showing the values of the array and auxiliary values after each iteration of the **while** loop in lines 4-11.

(2p)

2.2) Prove the following: When HOARE-PARTITION terminates, it returns a value j such that $p \leq j < r$.

(2p)

2.) Stooge sort
Professors Howard, Fine, and Howard have proposed the following ”elegant” sorting algorithm:

STOOGE-SORT (A, i, j)

1: **if** $A[i] > A[j]$ **then**
2: EXCHANGE $A[i] \leftrightarrow A[j]$
3: **end if**
4: **if** $i + 1 \geq j$ **then**
5: **return**
6: **end if**
7: $k \leftarrow \lfloor (j - i + 1)/3 \rfloor$
8: STOOGE-SORT $(A, i, j - k)$
9: STOOGE-SORT $(A, i + k, j)$
10: STOOGE-SORT $(A, i, j - k)$

2.1) Argue that, if $n = length[A]$, then STOOGE-SORT$(A, 1, length[A])$ correctly sorts the input array A[1..n]. Also give an example array (a small one) and write down the values of the array after the recurrence of STOOGE-SORT.

(2p)

2.2) Give a recurrence for the worst-case running time of STOOGE-SORT.

(2p)

Advanced Problem: Give a tight asymptotic (Θ-notation) bound on the worst-case running time. (This does not give extra points, but provides extra insight into the analysis of algorithms)